Name: $\qquad$ Date: $\qquad$
Notes
Algebra Section 11.2
Pages 719-726
Goal: "Simplify radicals using the product property" "Multiply radicals"
"Simplify radicals using the quotient property"
"Rationalize the denominator"
"Add and Subtract Radicals"

## Radicals are simplest form when:

1. The number under the $\qquad$ has no $\qquad$ .
2. No $\qquad$ have an $\qquad$ greater than 1.
3. There are no $\qquad$ under the radical sign.
4. There are no $\qquad$ in the $\qquad$ .

## Properties of Radicals

Product Property: $\sqrt{a \cdot b}=\sqrt{a} \cdot \sqrt{b}$ or $\sqrt{a} \cdot \sqrt{b}=\sqrt{a b}$ so....

$$
\sqrt{4 \cdot 9}=\sqrt{4} \cdot \sqrt{9}=2 \cdot 3=6
$$

Quotient Property: $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$ or $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}} \quad$ so....

$$
\sqrt{\frac{9}{16}}=\frac{\sqrt{9}}{\sqrt{16}}=\frac{3}{4}
$$

Examples (Simplifying):
$\sqrt{8}=\sqrt{4} \cdot \sqrt{2}$
$2 \sqrt{2}$
(Find the square root of the perfect square)
Try These:
$\sqrt{50}$
$\sqrt{12}$
$\sqrt{27}$
$\sqrt{32}$
$\sqrt{28}$
$\sqrt{48}$
$\sqrt{72}$
$\sqrt{80}$
$\sqrt{162}$
$\sqrt{600}$

Examples (variables):

$$
\begin{array}{ccc}
\sqrt{x^{2}} & x \cdot x=x^{2} & \sqrt{a^{2}} \\
x & a
\end{array}
$$

Try These:

$$
\sqrt{y^{2}}
$$

Examples (Multiplication):

$$
\sqrt{9 x^{2}}=\sqrt{9} \cdot \sqrt{x^{2}}=3 x
$$

$$
\sqrt{18 x^{2}}=\sqrt{9} \cdot \sqrt{2} \cdot \sqrt{x^{2}}=3 x \sqrt{2}
$$

Try These:
$\sqrt{16 x^{2}}$
$\sqrt{4 x^{2}}$
$\sqrt{49 x^{2}}$
$\sqrt{27 a^{2}}$
$\sqrt{20 b}$
$\sqrt{64 x^{2}}$
$\sqrt{8 x}$
$\sqrt{81 x^{2}}$
$\sqrt{45 x^{2}}$
$\sqrt{12 x^{2} y}$

Examples (Multiplication):
$\sqrt{6} \cdot \sqrt{6}=\sqrt{36}=6$

$$
4 \sqrt{x} \cdot \sqrt{3 x}=4 \sqrt{3 x \cdot x}=4 \cdot \sqrt{3} \cdot \sqrt{x^{2}}=4 x \sqrt{3}
$$

Try These:
$\sqrt{2} \cdot \sqrt{8}$
$\sqrt{20} \cdot \sqrt{5}$
$\sqrt{5 x} \cdot 3 \sqrt{x}$
$\sqrt{2} \cdot \sqrt{12}$
$2 \sqrt{2} \cdot 5 \sqrt{2}$
$4 \sqrt{3} \cdot 2 \sqrt{2}$
$7 \sqrt{3} \cdot 2 \sqrt{3}$
$4 \sqrt{5} \cdot 2 \sqrt{10}$

Examples (Division):
$\sqrt{\frac{13}{100}}=\frac{\sqrt{13}}{\sqrt{100}}=\frac{\sqrt{13}}{10}$

$$
\sqrt{\frac{7}{x^{2}}}=\frac{\sqrt{7}}{\sqrt{x^{2}}}=\frac{\sqrt{7}}{x}
$$

Try These:
$\sqrt{\frac{3}{9}}$
$\sqrt{\frac{5}{n^{2}}}$
$\sqrt{\frac{a}{b^{2}}}$
$\sqrt{\frac{w}{144}}$

## Rationalize the Denominator:

Radicals in the denominator (not perfect square).
Examples:
$\frac{3}{\sqrt{7}} \quad$ Multiply by $\frac{\sqrt{7}}{\sqrt{7}} \quad \frac{3}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}=\frac{3 \sqrt{7}}{\sqrt{49}}=\frac{3 \sqrt{7}}{7}$
$\frac{\sqrt{5}}{\sqrt{2 m}} \quad$ Multiply by $\frac{\sqrt{2 m}}{\sqrt{2 m}} \quad \frac{\sqrt{5}}{\sqrt{2 m}} \cdot \frac{\sqrt{2 m}}{\sqrt{2 m}}=\frac{\sqrt{10 m}}{\sqrt{4 m^{2}}}=\frac{\sqrt{10 m}}{2 m}$
Try These:
$\frac{1}{\sqrt{5}}$
$\frac{1}{\sqrt{x}}$
$\frac{2}{\sqrt{3}}$
$\frac{5}{\sqrt{7}}$
$\frac{\sqrt{2 a}}{\sqrt{a}}$
$\frac{3}{\sqrt{2}}$
$\frac{2}{\sqrt{3}}$
$\frac{5}{\sqrt{x}}$
$\frac{4}{\sqrt{n}}$
$\frac{\sqrt{a}}{\sqrt{2}}$
$\frac{1}{\sqrt{5}}$
$\frac{3}{\sqrt{x}}$
$\frac{2}{\sqrt{x}}$
$\frac{5}{\sqrt{6}}$
$\frac{\sqrt{2 a}}{\sqrt{8}}$

## Add and Subtract Radicals:

Radicals are like terms when: when the number under the radical sign (The radicand) is exactly the same. Combine like radical terms by adding or subtracting the coefficient.

Examples:
$3 \sqrt{5}+7 \sqrt{5}$
$10 \sqrt{5}$

$$
\begin{aligned}
& 4 \sqrt{10}+\sqrt{13}-9 \sqrt{10} \\
& 4 \sqrt{10}-9 \sqrt{10}+\sqrt{13} \\
& -5 \sqrt{10}+\sqrt{13}
\end{aligned}
$$

$$
\begin{gathered}
5 \sqrt{3}+\sqrt{48} \\
5 \sqrt{3}+\sqrt{16 \cdot 3} \\
5 \sqrt{3}+4 \sqrt{3} \\
9 \sqrt{3}
\end{gathered}
$$

Try These:
$2 \sqrt{3}+4 \sqrt{3}$
$\sqrt{6}+2 \sqrt{6}+3 \sqrt{6}$
$7 \sqrt{5}-2 \sqrt{5}$
$2 \sqrt{5}-8 \sqrt{5}$
$2 \sqrt{2}+\sqrt{8}$
$4 \sqrt{3}+2 \sqrt{27}$
$7 \sqrt{14}+\sqrt{21}-4 \sqrt{14}$
$2 \sqrt{7}+3 \sqrt{63}$
$2 \sqrt{7}+\sqrt{28}$

Distribute: (combine like terms if possible)
Example:
$\sqrt{5}(4-\sqrt{20})$
$4 \sqrt{5}-\sqrt{100}$
$4 \sqrt{5}-10$

$$
\begin{gathered}
(3 \sqrt{2})^{2}=3 \sqrt{2} \cdot 3 \sqrt{2} \\
9 \sqrt{4} \\
\\
9 \cdot 2=18
\end{gathered}
$$

Try These:
$\sqrt{3}(2+\sqrt{12})$
$3(\sqrt{2}-3 \sqrt{5})$
$\sqrt{2}(3+\sqrt{2})$
$(4 \sqrt{3})^{2}$
$(2 \sqrt{5})^{2}$
$\sqrt{2}(3 \sqrt{2}+7)$

Mixed Practice:
Simplify:
$\sqrt{98}$
$\sqrt{18 x^{2}}$
$\sqrt{4 a^{2}}$
$\sqrt{20 b}$

Multiply and Simplify:

$$
\begin{array}{lccc}
\sqrt{3} \cdot \sqrt{6} & \sqrt{5} \cdot \sqrt{20} & \sqrt{2} \cdot \sqrt{10} & 3 \sqrt{2} \cdot 4 \sqrt{8} \\
3 \sqrt{8} \cdot \sqrt{3} & \sqrt{6} \cdot 2 \sqrt{8} & 2 \sqrt{2} \cdot 3 \sqrt{6} & 6 \sqrt{3} \cdot 4 \sqrt{5} \\
3(2 \sqrt{2}-5) & \sqrt{2}(3 \sqrt{6}+\sqrt{24}) & (2 \sqrt{6})^{2}
\end{array}
$$

Divide and Simplify

$$
\begin{array}{lllll}
\sqrt{\frac{1}{4}} & \sqrt{\frac{2}{n^{2}}} & \sqrt{\frac{5}{x^{2}}} & \sqrt{\frac{6}{900}} & \\
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{b}} & \frac{5}{\sqrt{x}} & \frac{7}{\sqrt{3}} & \frac{\sqrt{5 a}}{\sqrt{a}}
\end{array}
$$

Add and Subtract:

$$
\begin{array}{lll}
3 \sqrt{5}+2 \sqrt{5} & \sqrt{7}+4 \sqrt{7}-3 \sqrt{7} & 2 \sqrt{3}-8 \sqrt{3} \\
2 \sqrt{8}+\sqrt{32}-4 \sqrt{12} & 4 \sqrt{6}+3 \sqrt{24} & 2 \sqrt{5}+\sqrt{500}
\end{array}
$$

